PLASTIC DEFORMATION OF METALS SUBJECTED TO INTENSIVE SOURCES

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The elastoplastic strain of metals being formed when they melt under the effect of a point heat source with a pulse duration greater than 10^{-6} sec is considered in this paper. The time development of the plastic strain and pressure domains in the melt is investigated. It is shown that two plastic strain domains occur during the interaction under consideration: a relatively broad domain of "mechanical" influence and a narrow domain of "thermal" influence. The stress–strain distributions as well as the hydrostatic pressure in the fluid are determined by a quasistationary temperature distribution starting with times corresponding to half (of the quasistationary) the value of the melt radius $X \sim 0.5$. It is shown that the dimensions of the weak and strong plastic strain domains formed by heat and acoustic waves grow continuously to the quasistationary values, while the hydrostatic pressure in the fluid reaches the maximum value for $X \sim 0.3...0.4$. The ratio between the radii of the plastic strain zones and of the liquid bath for a quasistationary temperature distribution in the first domain lies within the range 10-50, and does not exceed 1.7 for Cu, Ni, and Fe in the second. The anomalous nature of the development of the strong plastic strain domain in Al, because of migration of the metal grain boundaries to result in "collapse" of the domain for the values $X \sim 0.5$ accompanied by a jumplike diminution in the hydrostatic pressure in the fluid, is noted.

Traces of strong plastic strain [1, 2] have been detected on metal surfaces in the domain close to the melt subjected to highly intensive pulsed heat sources. It has hence been noted that the magnitude of the hydrostatic pressure in the melt is 10^3-10^6 atm [1], which substantially exceeds the yield point of the majority of metals. Therefore, it is necessary to consider the elastoplastic deformation scheme in solving the problem of metal strain under the effect of a pulsed heat source.

An attempt has been made in [3] to estimate the contribution of plastic strains in the melting of a metal by an electron beam focused under the metal surface, where it was assumed that the kinetics of melting can be described in the approximation of a quasistationary temperature distribution. The presence of one plastic strain phase was indicated in this paper, while two zones are detected experimentally [2], which differ essentially in the concentration of dislocations. Moreover, a stationary temperature approximation does not take account of the propagation velocity of the liquid phase boundary and does not yield a representation about the development of strains at the initial instants.

An analytical solution of the self-consistent Stefan problem in the presence of elastoplastic strain of the solid phase is fraught with great mathematical difficulties. Hence, it is interesting to solve model problems, whose results permit at least a qualitative examination of the mechanism of this complex process.

An idealized problem of the plastic deformation of metals subjected to a point heat source whose intensity does not exceed 10^6 W/cm^2 and which is within the bulk of the metal specimen is considered. The limitation on the source intensity permits considering the main process in treating the material to be melting and neglecting the contribution of different hydrodynamic effects. Let the depth at which the heat source lies satisfy the condition $L \gg V_S \tau$, where V_S is the speed of sound in the material, and τ is the time of pulse action. This permits considering spherical symmetry conserved in both the heat-conduction and the deformation problems.

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TABLE 1

Metal	Parameters of (8), (9)			Roots of (9)	Corresponding value of pressure (8)	
	$\frac{\alpha T_m E}{(1-\nu)\sigma_s^0}$	$\frac{\varepsilon_0 E}{(1-\nu)\sigma_s^0}$	$\frac{3\alpha T_m}{2\varepsilon_0}$		P/σ_s^0	P, atm
Cu	40.5	35	1.95	21	6	$4.2\cdot10^3$
				1.28	21.2	$1.5\cdot10^4$
				−1.23	—	—
Ni	18.0	15	2.4	10	4.6	$1.4\cdot10^3$
				1.6	8.0	$2.4\cdot10^3$
				−1.35	—	—
Fe	93	57	2.55	48	7.8	$3.9\cdot10^3$
				1.38	45	$2.3\cdot10^4$
				−1.33	—	—
Al	42	59	1.1	22	6.2	$2.5\cdot10^3$
				0.52	—	—
				−0.52	—	—

The temperature distribution in the solid phase depends strongly on the forward velocity of the phase interface $\dot{X} = (1-X)/X^2$, where X is the dimensionless radius of the melt [4], and causes a stress state which varies with the time. The distribution of the thermoelastic stresses is quasistationary in the time interval in which $V_s \gg \dot{X}$, i.e., is time-dependent in terms of a parameter. It can be shown that the problem of thermoelastic stresses and strains can be considered in a quasistationary approximation in the stresses and strains when

$$X \gg (2A)^{-1}(\sqrt{1+4A} - 1), \tag{1}$$

where $A = V_s\alpha^*/\beta^2$, $\alpha^* = F/2\pi Q\gamma$, $\beta = \lambda(T_m - T_0)/Q\gamma$ are constants; F is the power of the heat source, W; Q is the specific heat of melting; cal/g; γ is the specific gravity of the material, g/cm^3; λ is the coefficient of thermal conductivity, cal/cm · sec · °C; T_m is the melting point, °C; and T_0 is the temperature of the ambient medium, °C. For intensities on the order of 10^5-10^6 W/cm^2 we have $A \gg 1$. Then the inequality (1) is represented as $X \geq A^{-1/2}$, which corresponds to times on the order of $(\alpha^*)^{1/2}V_s^{-3/2}$ sec, when dimensionality is taken into account. For the majority of metals under consideration this quantity does not exceed 10^{-8} sec for a 100-W power source, which allows quasistationary analysis of the problem of thermoelastic stresses and strains originating under the effect of a heat source with a pulse duration on the order of fractions of a microsecond and more. Appropriate values of the melt radius do not exceed $X_1 = 0.1$.

Let us consider a semiinfinite solid in which the boundary separating the liquid and solid phases has the dimension $X(\xi)$ (in the units α^*/β) at a time ξ [in the units $(\alpha^*)^2/\beta^3$], and the boundary separating the plastic (adjoining the liquid) and elastic domains of the solid has the dimension $Y(\xi) \geq X(\xi)$. Let us use the scheme of an elastoplastic body to find the thermoelastic stresses and the scheme of an ideally plastic body to describe the plastic properties of the substance.

By virtue of spherical symmetry, the radial σ_r and tangential σ_t components of the stress tensor and the corresponding components ε_r and ε_t of the strain tensor are different from zero. Hence, σ_r and σ_t are connected by the equilibrium condition, and ε_r and ε_t by the compatibility condition [5]:

$$d\sigma_r/dR - 2(\sigma_t - \sigma_r)/R = 0; \tag{2}$$

$$d/dR(R\varepsilon_t) - \varepsilon_r = 0. \tag{3}$$

The boundary conditions are [6]

$$\begin{gathered} \sigma_r|_{R=X(\xi)} = -P; \\ \sigma_r|_{R\to\infty} = 0; \\ \varepsilon_t|_{R=X(\xi)} = \varepsilon_0 - k_1P, \end{gathered} \tag{4}$$

where P is the hydrostatic pressure, atm, which is constant over the volume of the melted metal; ε_0 is the change in specific gravity of the substance during melting; and k_1 is the coefficient of fluid compressibility. The stresses, strains, and displacements should be continuous on the boundary between the elastic and plastic domains; the plasticity conditions

$$(\sigma_t - \sigma_r) = \sigma_s(R),\ X(\xi) \leqslant R \leqslant Y(\xi) \tag{5}$$

should be satisfied in the plastic domain.

It should be emphasized that because of the presence of a temperature gradient in the solid phase, the yield point $\sigma_s(R)$ of the substance is a function of the temperature or the running radius, and its value on the boundary between the elastic and plastic domains σ_s^Y differs from the yield point σ_s^0 at a normal temperature. Let us use the dependence of the yield point near the melting point [3] by taking account of the dependence of the temperature distribution in the solid phase in the case of a quasistationary temperature distribution [4]

$$T(R,\ X(\xi)) = T_m X(\xi)/R.$$

Then $\sigma_s(R)-\sigma_s^0(T_m-T_0)/T_m=\sigma^0(1-X(\xi)/R)$ and σ_s^Y can be written as $\sigma_s^Y=\sigma_s^0(1-X(\xi)/Y(\xi))$. Taking account of (5) and the connection between the stresses and strains in the elastic domain

$$\begin{cases} \varepsilon_r = [\sigma_r - 2\nu\sigma_t]/E + \alpha T, \\ \varepsilon_t = [(1-\nu)\sigma_t - \nu\sigma_r]/E + \alpha T, \end{cases} \tag{6}$$

where ν is the Poisson ratio; E is the elastic modulus, atm; and α is the linear coefficient of expansion of the substance, $(^\circ\mathrm{C})^{-1}$, (2) and (3) result in the following equations:

elastic domain $Y(\xi) \leq R < \infty$

$$\begin{cases} d\sigma_r/dR - 2(\sigma_t - \sigma_r)/R = 0, \\ d\sigma_t/dR + (\sigma_t - \sigma_r)/R = -E\alpha/(1-\nu)\cdot dT/dR; \end{cases} \tag{7}$$

plastic domain $X(\xi) \leq R \leq Y(\xi)$

$$d\sigma_r/dR = 2\sigma_s(R)/R.$$

It can be shown that the radial and tangential components of the stress and strain tensors in the elastic domain, which are a solution of (7), are determined by the elastic and plastic components. These latter can be neglected in practice, since the yield point at normal temperature σ_s^0 is two–three orders less than the elastic modulus for the majority of metals. The magnitude of the elastic components is determined by the temperature gradient in the solid phase. Using the stress continuity condition on the boundary between the elastic and plastic domains, we obtain an expression for the hydrostatic pressure in the fluid:

$$\frac{P}{2\sigma_s^0} = -2\ln\frac{X(\xi)}{Y(\xi)} - \frac{4}{3} + \left[\frac{2E\alpha T_m}{3(1-\nu)\sigma_s^0} + \frac{4}{3}\right]\frac{X(\xi)}{Y(\xi)}. \tag{8}$$

Since it has been observed experimentally that the pressure in the fluid exceeds the yield point at normal temperature by several tens of times, then the pressure is evidently determined mainly by the last member in (8). The numerical value of $P/2\sigma_s^0$ depends on the ratio between the radii of the fluid drop and the plastic strain zone. Therefore, the hydrostatic pressure in the fluid is characterized for elastoplastic strain by the product of the ratio between the elastic modulus and the yield point σ_s^0 and a quantity characterizing the increase in the linear dimensions during heating to the melting point, or the quantity $2E\alpha T_m/3(1-\nu)\sigma_s^0$. It is interesting to note that the expression for the pressure in the fluid during pure elastic strain agrees with this same value, but the product αT_m must hence be replaced by the value of the change in the specific gravity of the substance during melting ε_0.

Therefore, the quasistationary problem of elastoplastic strains turns out to be statically determinate, since the pressure in the fluid, the stress and strain distribution in the elastic domain, and also the stress distribution in the plastic domain are determined in terms of the parameter $Y(\xi)$ without the need to find the strain distribution in the plastic zone. To determine $Y(\xi)$, let us find the strain distribution in the plastic domain. Under the assumption that the strain is small ($\varepsilon_i \ll 1$), the volume mechanical properties of the elastic and plastic zones are assumed identical [7]. In general form the volume expansion of a solid is described as

$$3\varepsilon = \varepsilon_r + 2\varepsilon_t = k_2(\sigma_r + 2\sigma_t) + 3\alpha T,$$

where $k_2=(1-2\nu)/E$ is the coefficient of compressibility of the solid.

Using (2), the relationships between the strains and displacements $\varepsilon_t=U/R$, $\varepsilon_r=dU/dR$ [where $U(R)$ is the displacement in the units α^*/β], and also the continuity condition for the displacements on the boundary of the elastic and plastic domains, let us write the expression for the displacements as

$$U(R) = k_2 R\sigma_r + \frac{\alpha T_m}{2}X(\xi)\left[3 - \frac{Y(\xi)^2}{X(\xi)^2}\right] + \frac{\sigma_s^0(1-\nu)}{E}\left[1 - \frac{X(\xi)}{Y(\xi)}\right]\frac{Y^3(\xi)}{R^2},$$

from which we obtain by using (4)

$$\varepsilon_0 + (k_2 - k_1)P = \frac{\alpha T_m}{2}\left[3 - \frac{Y(\xi)^2}{X(\xi)^2}\right] + \frac{\sigma_s^0(1-\nu)}{E}\left[1 - \frac{X(\xi)}{Y(\xi)}\right]\frac{Y(\xi)^3}{X(\xi)^3}.$$

Assuming the compressibility coefficients of the liquid and solid phases to be equal $k_1 = k_2$, we obtain a cubic equation to determine the ratio $Y(\xi)/X(\xi)$:

$$\frac{Y(\xi)^3}{X(\xi)^3} - \left[\frac{\alpha T_m E}{2(1-\nu)\sigma_s^0} + 1\right]\frac{Y(\xi)^2}{X(\xi)^2} + \frac{\varepsilon_0 E}{(1-\nu)\sigma_s^0}\left(\frac{3\alpha T_m}{2\varepsilon_0} - 1\right) = 0. \tag{9}$$

Since the values $E/\sigma_s^0 \sim 350$ [7], $\alpha T_m \sim 2\cdot 10^{-2}$, $3\alpha T_m/2\varepsilon_0 \sim 1.5$, hold for the majority of metals, and ν does not emerge beyond the limit 0.3-0.5 in the temperature range from zero to T_m, the approximate equation for the ratio $Y(\xi)/X(\xi)$ can then be written in general form as

$$Y(\xi)^3/X(\xi)^3 - 6.8Y(\xi)^2/X(\xi)^2 + 5.8 = 0, \tag{10}$$

and the magnitude of the pressure in the fluid as

$$P/2\sigma_s^0 \sim 2\alpha T_m E/3(1-\nu)\sigma_s^0 \cdot X(\xi)/Y(\xi) \sim 7.8X(\xi)/Y(\xi).$$

It can be seen that (10) has three real roots: 1; 6, 7; −0.85, from which the first two turn out to be acceptable from physical considerations. Since the problem is practically transformed into a purely elastic problem for $Y(\xi)/X(\xi) \sim 1$, the single approximate value of the ratio $Y(\xi)/X(\xi)$ turns out to be the ratio 6.7, to which the following hydrostatic pressure corresponds:

$$P/\sigma_s^0 \sim 7.8/6.7 = 1.2.$$

Therefore, a plastic strain domain, whose dimensions exceed the melt radius several-fold, is formed for a quasistationary temperature distribution in the solid phase subjected to a pressure on the order of the material yield point. This agrees with the estimate of the plastic strain domain in a semiinfinite specimen subjected to the effect of just hydrostatic pressure (without a temperature gradient). In this case the size of the plastic domain is estimated to be 5-6 radii of the spherical cavity and the pressure needed to form it to be several magnitudes of σ_s^0 [7].

The coefficients of (9) for different metals can be determined by using the data in Table 1. The solution of the appropriate equations shows that (9) has two roots which do not contradict the physical sense for the majority of metals. The first and greatest root, equal in order of magnitude to

$$\frac{\alpha T_m E}{2(1-\nu)\sigma_s^0} + 1 \approx \frac{\alpha T_m E}{2(1-\nu)\sigma_s^0}, \tag{11}$$

is determined by the elastic properties of the material and has values on the order of several tens, which corresponds to a broad plastic strain zone whose boundary is found for very low temperatures equal to 5, 10, 2, and 5% of T_m for Cu, Ni, Fe, and Al, respectively. Therefore, the root (11) characterizes the zone of "mechanical" influence and corresponds to a "weak" plastic strain wave detached from the phase-transition boundary, in which the yield point of the material $\sigma_s(T)$ actually corresponds to the yield point at room temperature σ_s^0. The pressure in the melt hence turns out to be equal to several magnitudes of σ_s^0. The second root, equal in order of magnitude to

$$\left[\frac{\varepsilon_0 E(3\alpha T_m/2\varepsilon_0 - 1)}{(1-\nu)\sigma_s^0(\alpha T_m E/2(1-\nu)\sigma_s^0 + 1)}\right]^{1/2} \approx \left(3 - \frac{2\varepsilon_0}{\alpha T_m}\right)^{1/2}, \tag{12}$$

is determined by the ratio between the change in the specific gravity of the substance during melting ε_0 and the increase in the linear dimensions during heating to T_m and has values somewhat exceeding one, which corresponds to a narrow plastic strain zone or a zone of "thermal" influence whose boundary is at significant temperatures equal to 78, 63, and 72% of T_m for Cu, Ni, and Fe, respectively. In this domain the yield point of the material $\sigma_s(T)$ is considerably less than the yield point at normal temperature σ_s^0 because of the significant temperature gradient. The pressure in the melt is hence 10-50 magnitudes of σ_s^0. Aluminum has a somewhat greater value of ε_0 compared to the other materials, and there is practically no "strong" plastic strain domain for a quasistationary temperature distribution therein. The sharp distinction between the nature of plastic strain in aluminum from that in other metals can be explained by the strong migration of the grain boundaries at high temperatures, hindering the development of a high stress concentration [8]. As is seen from (12), the maximum value of the root $Y(\xi)/X(\xi)$ governing the size of the "thermal" influence domain 1.7, corresponds to the case when no change occurs in the volume during melting ($\varepsilon_0 = 0$).

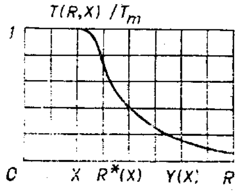

Fig. 1

Therefore, a sufficiently broad domain of "weak" plastic strain occurs for quasistationary melting, in which the pressure is several magnitudes of σ_s^0. A narrow "strong" plastic strain domain occurs near the melt boundary for the majority of metals, where the yield point is substantially less than σ_s^0 and the pressure in the melt exceeds σ_s^0 several tens of times. The total pressure in the fluid is the sum of these two pressures. The relative size of the plastic strain zone is independent of the source intensity. However, the absolute value of the radius of the plastic strain domain depends on the source intensity in terms of the parameter $X(\xi)$ in the units $F/2\pi\lambda(T_m-T_0)$ [4].

Let us consider the development of stress and strain with time by using the temperature distribution in the solid phase [4] corresponding to the Stefan self-consistent problem with a point heat source:

$$T(R, X) = (1-\varkappa)(\eta\dot{X}R)^{-1}\{1-\exp[-\eta\dot{X}(R-X)]\}, \tag{13}$$

where R and X are the running radius and the radius of the fluid drop in the units $\alpha^*/\beta, \eta = \beta/a$, $\varkappa = T_0/T_m$, a is the coefficient of temperature conduction of the solid phase, cm^2/sec.

The expression for the temperature (13) has been obtained in the range of distances $X \le R \le R^*(X)$, where $R^*(X)$ corresponds to the boundary of the heat-wave front (in units α^*/β) and is determined from the condition $R^*(X)=X(2-X)$. For greater distances $R \ge R^*(X)$ the temperature distribution can be described by a dependence of the form $T(R, X) = T^*(R^*, X)R^*/R$ which assures continuity of the temperature on the heat-wave front, where

$$T^*(X)/T_m = 1-[(1-\varkappa)/\eta]\,[X/(1-X)(2-X)]\ \left[1-e^{-\eta(1-X)^2/X}\right]. \tag{14}$$

Since $\varkappa = T_0/T_m \le 1$, the temperature distribution in the solid phase is representable as

$$T(R, X)/T_m = 1-[\delta(X)/R][1-e^{-(R-X)/\delta(X)}],\ X \leqslant R \leqslant R^*(X); \tag{15}$$

$$T(R, X)/T_m = [T^*(R^*, X)/T_m]R^*(X)/R = \Delta_1 X/R,\ R^*(X) \leqslant R < \infty, \tag{16}$$

where $\delta(X) = (\eta\dot{X})^{-1} = \eta^{-1}X^2/(1-X)$; $\Delta_1 = T^*(R^*, X)/T_m \cdot R^*(X)/X$. It should be emphasized that the temperature distribution in the form (15), (16) is considered starting with times corresponding to values of the melt radius equal to 0.1. As is seen from (15), as the melt radius tends to the quasistationary value 1, the quantity $\delta(X) \to \infty$ and (15) becomes

$$T(R, X)/T_m = (X/R)[1+0{,}5[X/\delta(X)](R/X-1)^2],\ X \leqslant R \leqslant R^*(X),\quad X \to 1. \tag{17}$$

By using (17) we estimate the value of the melt radius for which the second member in (17) is much less than one and the temperature distribution over the whole solid phase is quasistationary. It can be shown that this condition corresponds to the values $X_2 \ge 0.5\eta/(1+0.5\eta)$. Appropriate values of X_2 are 0.41; 0.49; 0.51; 0.67 for Al, Cu, Ni, and Fe, respectively.

Since the boundary of the zone of acoustic influence exceeds the boundary of the heat-wave front $R^*(X)$ even for small times of action on the order of 10^{-6} sec, then the plastic strain domain with boundaries $Y(X)$ (in the units α^*/β) is representable as two domains, in one of which (located beside the elastic domain), $T(R,X)$ is determined by (16), and in the other (adjoining the melt zone), by (15), (17). The temperature distribution in the solid phase is represented in Fig. 1, where $X(t)$ is the boundary of the liquid bath, $R^*(X)$ is the boundary of the heat-wave front, and $Y(X)$ is the boundary of the plastic strain domain. Then the plasticity condition can be written as

$$(\sigma_t-\sigma_r) = \sigma_s(R) = \sigma_s^0\cdot\delta(X)/R\cdot\left[1-e^{-(R-X)/\delta(X)}\right],\quad X \leqslant R \leqslant R^*(X); \tag{18}$$

$$(\sigma_t-\sigma_r) = \sigma_s(R) = \sigma_s^0\left\{1-\frac{X}{R}-0{,}5\frac{X}{\delta(X)}\left[\frac{R}{X}-2-\frac{X}{R}\right]\right\}.\quad X \leqslant R \leqslant R^*(X),\quad X \to 1; \tag{19}$$

$$(\sigma_t - \sigma_r) = \sigma_s(R) = \sigma_s^0\left[1 - \frac{T^*(R^*, X)}{T_m}\frac{R^*(X)}{R}\right], \qquad R^*(X) \leqslant R \leqslant Y(X), \tag{20}$$

where the value of the yield point on the boundary of the elastic and plastic domains σ_S^Y equals $\sigma_S^0(1 - T^* R^* / T_m Y)$ and does not agree with the yield point at a normal temperature σ_S^0 because of the presence of the temperature gradient in the solid phase. The σ_t and σ_r in (18)-(20) denote the tangential and radial stresses, respectively.

The elastic domain is $Y(X) \leq R < \infty$. In this domain σ_r and σ_t, related by the equilibrium condition, and the radial ε_r and tangential ε_t strains related by the compatibility condition, yield the system of equations (7). In addition, known relationships between the stresses and the strains (6) as well as condition (20) for $R = Y(X)$ and the second condition in (4) hold.

The plastic domain is $X \leq R \leq Y(X)$. To find the stresses in this domain, let us use the first equation in (7) while taking account of (18)-(20) and the continuity conditions for the stresses on the boundary of the elastic and plastic domains, as well as the plastic stress domains with a different temperature distribution. Moreover, conditions (4) should be satisfied on the boundary with the liquid phase.

The solution of the equations for the stresses in the elastic and plastic domains results in expressions for the radial stresses σ_r in the plastic strain domains $R^*(X) \leq R \leq Y(X)$ with a temperature distribution of the form (16) and $X \leq R \leq R^*(X)$ (for two time intervals $X_1 = 0.1 \leq X \leq X_2$ and $X_2 \leq X \leq 1$) with a temperature distribution of the form (15), (17), from which it is seen that they are determined by three components. The first are due to thermoelastic stresses generated in the elastic strain domain; the second, by plastic stresses in the domain $R^*(X) \leq R \leq Y(X)$ which are independent of the forward velocity of the phase interface; and the third, by plastic stresses in the domain $X \leq R \leq R^*(X)$, which are dependent on the forward velocity of the boundary of the heat-wave front. The latter components tend to zero as $X \to 1$ and $X_1 = 0.1$. The main contribution to the value of the stress is introduced by the first components, in which the term

$$\frac{E\alpha T^*}{3(1-\nu)\sigma_s^0}\frac{R^*(X)}{Y(X)} = \frac{E\alpha T_m}{3(1-\nu)\sigma_s^0}\frac{X}{Y(X)}\Delta_1$$

plays the main role, where the quantity Δ_1 agrees with that in (16), and $Y(X)/X$ is the ratio between the sizes of the plastic strain and the melting zones.

To find the changes in the hydrostatic pressure values in the liquid phase with time, let us use the first equation in (4) by equating it to the expressions obtained for the stresses in the plastic strain domains with a different temperature distribution. Then

$$\frac{P}{2\sigma_s^0} = \Delta_2 + \ln\frac{Y(X)}{X} + \frac{\Delta_1}{3}\left[\frac{E\alpha T_m}{(1-\nu)\sigma_s^0} + 2\right]\frac{X}{Y(X)}, \tag{21}$$

where

$$\Delta_2 = [\delta(X)/X](1 - X/R^*(X)) - \ln[R^*(X)/X] - T^*(R^*, X)/T_m + 1/3,$$
$$X_1 = 0.1 \leqslant X \leqslant X_2;$$
$$\Delta_2 = (1 - X/R^*(X))\,[1 - 0.5[X/\delta(X)](1 + R^*(X)/X)] - [X/\delta(X)] \times$$
$$\times \ln[R^*(X)/X] + 1/3 - T^*(R^*, X)/T_m, \quad X_2 \leqslant X \leqslant 1.$$

As is seen from (21), the thermoelastic stresses introduce the main contribution to the magnitude of the hydrostatic pressure in the fluid $P/2\sigma_S^0$ and $\Delta_2 \to -2/3$ for $X \to 1$ and (21) goes over into the equation for the quasi-stationary temperature distribution (8). At a fixed time the value of $P/2\sigma_S^0$ is determined by the product of the coefficient Δ_1 characterizing the temperature distribution behind the heat-wave front, and the ratio between the sizes of the plastic strain and melt domains $Y(X)/X$. To determine the size of the plastic strain domain, let us find the strain distribution in the plastic strain domains with the different temperature distribution.

From the solution of the general equation for the displacements in the solid phase, obtained under the approximation of small strains [7]

$$dU/dR + 2U/R = k_2(3\sigma_r + R \cdot d\sigma_r/dR) + 3\alpha T,$$

we obtain the distribution of the tangential strains ε_t in the plastic strain domains $R^*(X) \leq R \leq Y(X)$ with a temperature distribution of the form (16), and $X \leq R \leq R^*(X)$ (for two time intervals $X_1 = 0.1 \leq X \leq X_2$ and $X_2 \leq X \leq 1$) with a temperature distribution of the form (15), (17). Furthermore, taking account of the third equation in (4), we obtain an equation to determine the ratio $Y(X)/X$ in the two time intervals $X_1 = 0.1 \leq X \leq X_2$ and $X_2 \leq X \leq 1$:

$$\frac{Y^3(X)}{X^3}-\Delta_1\left[\frac{E\alpha T_m}{2(1-\nu)\sigma_s^0}+1\right]\frac{Y^2(X)}{X^2}+\frac{\varepsilon_0 E}{(1-\nu)\sigma_s^0}\left[\Delta_{21}\frac{\alpha T_m}{\varepsilon_0}-1\right]=0, \tag{22}$$

$$\Delta_{21}=\left(\frac{3}{2}\Delta_1\frac{R^{*2}}{X^2}+1\right)-\frac{R^{*3}}{X^3}-\frac{3\delta(X)}{2X}\left(\frac{R^{*2}}{X^2}-1\right)-\frac{3\delta^2(X)}{X^2}\left[\left(1+\frac{\delta(X)}{X}\right)-\right.$$

$$\left.-\frac{R^*}{X}\left(1+\frac{\delta(X)}{R^*}\right)e^{-(R^*/X-1)/\delta(X)}\right],\quad X_1=0.1\leqslant X\leqslant X_2;$$

$$\frac{Y^3(X)}{X^3}-\Delta_1\left[\frac{E\alpha T_m}{2(1-\nu)\sigma_s^0}+1\right]\frac{Y^2(X)}{X^2}+\frac{\varepsilon_0 E}{(1-\nu)\sigma_s^0}\left[\frac{3}{2}\Delta_{22}\frac{\alpha T_m}{\varepsilon_0}-1\right]=0, \tag{23}$$

$$\Delta_{22}=\left[(\Delta_1-1)\frac{R^{*2}}{X^2}+1\right]+0.5\frac{X}{\delta(X)}\left[\frac{1}{6}-\frac{R^{*2}}{X^2}\left(1-\frac{4R^*}{3X}+\frac{R^{*2}}{2X^2}\right)\right],$$

$$X_2\leqslant X\leqslant 1.$$

It is seen from this last expression that Δ_{22} is practically equal to one during the time interval $X_2 \leq X \leq 1$; hence, the ratio Y(X)/X is constant in this range, and (23) goes over into the equation for a quasistationary temperature distribution (9) as X → 1. The cubic equations (22), (23) have two roots which do not contradict the physical meaning. The first and greater root corresponds to acoustic wave propagation, or propagation of a "weak" plastic strain wave, and has the form

$$Y(X)/X \sim \Delta_1 E\alpha T_m/2(1-\nu)\sigma_s^0,$$

from which it follows that the maximal ratio Y(X)/X corresponds to the initial times. Appropriate values of the sizes of the plastic strain zone Y(X) due to acoustic interaction are 2, 4, 9, and 4 for Ni, Cu, Fe, and Al, and the times of acoustic wave traversal of these distances for a source of 100-W power are $2\cdot10^{-8}$; $5\cdot10^{-9}$; $2\cdot10^{-7}$; and $2\cdot10^{-8}$ sec, respectively.

Therefore, the formation of a "weak" plastic strain domain is due to the thermoelastic stresses originating in the solid phase because of a temperature distribution of the form (16). Magnification of the size of the "weak" plastic strain domain (or the acoustic influence domain) is represented in Fig. 2 as a function of the radius of the liquid bath X(t) up to the quasistationary values of 21, 48, and 22 for Cu, Fe, and Al.

The second and smaller root corresponds to the heat wave, or "strong" plastic strain wave, and is expressed in terms of

$$Y(X)/X \sim (\Delta_1)^{-1/2}(2\Delta_{21}-2\varepsilon_0/\alpha T_m)^{1/2}; \tag{24}$$

$$Y(X)/X \sim (\Delta_1)^{-1/2}(3\Delta_{22}-2\varepsilon_0/\alpha T_m)^{1/2}. \tag{25}$$

As is seen from (24), (25), the formation of a "strong" plastic strain domain is due to the change in density of the substance during melting which occurs on the phase-transition boundary. The change in the boundary of the "strong" plastic strain domain (or the thermal influence domain) is represented in Fig. 3 as a function of the liquid bath radius X(t) for Al, Cu, and Fe. As is seen from Fig. 3, as the heat wave advances deep into the material, the "strong" plastic strain domain reaches the quasistationary state with radii 1.28 and 1.38 for Cu and Fe. However, the "strong" plastic strain domain reaches the maximum value for Al at X ~ 0.5 and not at X = 1. Then "collapse" of the "strong" plastic strong domain, accompanied by a jumplike diminution in the hydrostatic pressure in the fluid, seems to occur. The change in hydrostatic pressure in the liquid bath as a function of its radius X(t) is represented in Fig. 4 for Al, Cu, and Fe. The nature of the change in the "strong"

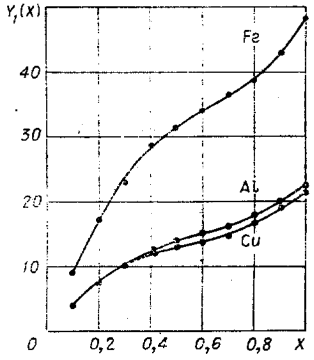

Fig. 2

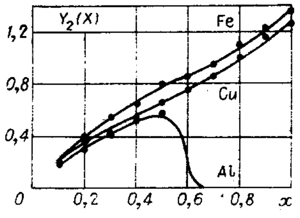

Fig. 3

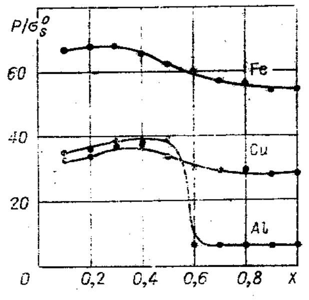

Fig. 4

plastic strain domain in Al can be explained by migration of the metal grain boundaries at high temperatures, which hinders the development of a high stress concentration in the metal [8].

The hydrostatic pressure in a fluid is evidently the sum of two pressures due to acoustic and thermal effects. A pressure on the order of several magnitudes of σ_S^0, which is generated because of thermoelectric stresses, does not change in practice during the whole period of liquid drop development, and the pressure due to a change in density of the substance during melting determines the nature of the pressure change in the liquid drop.

As is seen from Fig. 4, the nature of the change in hydrostatic pressure with time is dissimilar for different metals. However, the general tendency to pressure growth from the initial value corresponding to the melt radius $X_1 = 0.1$ to some maximum value corresponding to $X \sim 0.2$-0.5 is characteristic. The maximum pressure, equal to 24, 34, and 16 kbar, is built up at melt radii on the order of 0.4, 0.25, and 0.5 for Cu, Fe, and Al, respectively. The pressure increase is hence several yield points of the material at room temperature σ_S^0. Afterwards, the pressure in Cu and Fe drops smoothly to the quasistationary values of 15 and 23 kbar, which is less than the initial pressure corresponding to $X_1 = 0.1$. Since Al does not sustain high stress concentrations (on the order of several tens of the yield point at normal temperature) because of grain boundary migration, "collapse" of the "strong" plastic strain domain seems to occur with an appropriate pressure drop to the level characterizing the "weak" plastic strain domain and equal to 2.5 kbar.

Therefore, an estimate of the change in size of the plastic strain domain and of the appropriate values of the pressure with time showed that the maximum values of the hydrostatic pressure originate for melt radii on the order of 0.2-0.5. This is apparently related to the fact that this time is characterized by the maximum volume of melted substance responsible for the magnitude of the hydrostatic pressure.

The presence of two plastic strain domains with a different concentration of dislocations and degree of material hardening was observed experimentally during the effect of pulses of millisecond duration on iron [2]. A ratio $Y(\xi)/X(\xi)$ somewhat exceeding one has been obtained experimentally in investigations of the strain of beryllium monocrystals subjected to radiation with 30 J energies and 1 msec pulse durations [1]. An estimate of the size of the plastic strain domain for Fe is in satisfactory conformity with an experiment performed at a 500 W source intensity and 3.8 msec duration of the effect [2]. The experimental magnitude of the plastic strain zone is 0.55 and 1.08 mm for a source intensity of 430 and 1200 W. An appropriate computational estimate of $Y(\xi)$ is 0.52 and 1.10 mm. Therefore, the relationships obtained are in satisfactory qualitative correspondence with the results of experiments [1, 2].

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